$$\text{Then } x = \frac{\log(P+P_1)[(1+r)^t(1+r)^{t_1} - \log[P(1+r)^{t_1} + P_1(1+r)^t]}{\log(1+r)} \quad \ldots .(4).$$

But (4) and (3) being identical, either method may be used when compound interest is considered. The first, or correct, method by simple interest becomes very complicated when more than two payments are considered ; yet when we recall the fact that equation of payments is a subject of no practical importance, making approximate methods less desirous, it matters little how complicated the method may be if it is correct in theory.

The following method, which is found in most arithmetics is very often not much better than a good guess. A review of the solution will, at once, show the erroneousness of the method.

$P(t-x)r$=interest on P for $(t-x)$ years.

$P_1(x-t_1)r$=interest on P_1 for $(x-t_1)$ years.

$P(t-x)r = P_1(x-t_1)r$.

$$\therefore x = \frac{Pt+P_1t_1}{P+P_1}.$$

III. By Annual Interest.

$$\frac{Pr[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}{1+r[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]} = \text{discount on } P \text{ for } (t-x) \text{ years.}$$

$P_1r[(x-t_1)+\frac{1}{2}r(x-t_1)(x-t_1-1)]$=interest on P_1 for $(x-t_1)$ years.

$$\therefore \frac{Pr[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}{1+r[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]} = P_1r[(x-t_1)+\frac{1}{2}r(x-t_1)(x-t_1-1)] \quad \ldots ..(5).$$

From (5) x, the equated time, can be found.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from the August-September Number.]

Proposition XXVIII. *If two straights AX, BX (produced from any-sized straight AB toward the same parts, the first under an acute angle, and the other perpendicularly) mutually approach each other ever more without any certain limit, save at their infinite production ; I say all angles (Fig. 33.) at any points L, H, D of AX, from which are dropped to the straight BX perpendiculars LK, HK, DK,*

first will all be obtuse toward the parts of the point A, secondly will be ever less, the more distant from this point A, and finally the angles more and more distant from this same point A ever more without any certain limit approach to equality with a right angle.

Demonstratur. The first part follows indeed from Corollary I to Proposition XIII. The second part however is proved thus. For the two angles together at LK toward the base AB are greater (from Corollary to Proposition XVI.) than the two internal and opposite angles together at HK toward the same base AB.

But the angles at each point K toward the base AB are equal to each other, as being right. Therefore the obtuse angle at L toward the base AB is greater than the obtuse angle at H toward the same base AB.

In like manner is shown that the aforesaid obtuse angle at H is greater than the obtuse angle at the point D.

And thus ever, proceeding toward the points X.

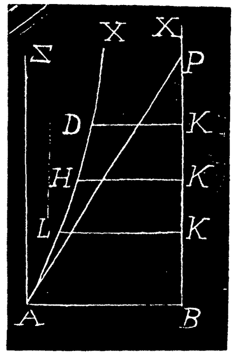

Fig. 33.

Finally the third part requires a longer disquisition. If therefore it can be done, let there be assigned (Fig. 34.) a certain angle MNC, than which is always greater, or anyhow not less, the excess of any of the aforesaid obtuse angles above a right angle. It follows (from Proposition XXI.) that the sides NM, NC comprehending that angle MNC can be so produced that the perpendicular MC from a certain point M of MN let fall upon NC may be greater (even in the hypothesis of acute angle) than any assigned finite length, as for instance the aforesaid base AB.

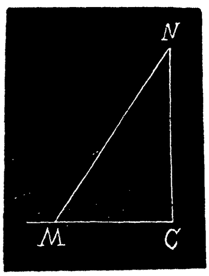

Fig. 34.

This standing; assume in BX (Fig. 35.) a certain BT equal to CN, and erect from the point T toward AX the perpendicular TS, which obviously (from Scholion to Proposition XXIV.) meets AX in a certain point S. Then from the point S let fall to AB the perpendicular SQ.

This falls (because of Euclid I. 17.) toward the parts of the acute angle between the points A and B. Again, acute will be the angle QST in the quadrilateral $QSTB$, since the remaining three angles are right; else (against Proposition V. and Proposition VI.) we come upon the hypothesis either of right angle or of obtuse angle.

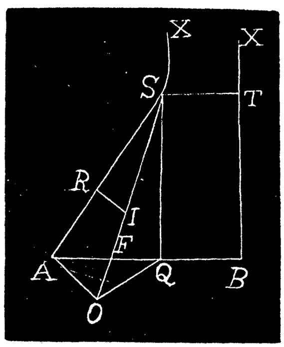

Fig. 35.

Hence the straight SQ will be greater (from Corollary I. to Proposition III.) than the straight BT,

or CN ; and again the angle ASQ will be greater than the excess by which the obtuse angle AST exceeds a right angle, and thus greater than the angle MNC. Draw therefore a certain SF cutting AQ in F and making with SA an angle equal to MNC. Then from the point A draw to SF produced the perpendicular AO. The point O falls (from Euclid I. 17.) below the point F, since the angle AFS (by Euclid I. 16.) is obtuse.

Finally, however ; since FS is greater (by Euclid I. 19.) than QS and so much greater than BT or CN, assume in FS the piece IS equal to CN, and from the point I erect to FS the perpendicular IR meeting AS in the point R.

But the point R falls between the points A and S : for if it fell on any point of AF, we would have in the same triangle (against Euclid I. 17.) two angles greater than two right angles, since the angle at the point F toward the parts of the point A has already been shown obtuse.

After so much preparation thus I conclude. Since in the quadrilateral $AOIR$ the angles at the points O and I are right, and the angle at the point A (by Euclid I. 17.) is acute because of the right angle AOS, and again the angle IRA (by Euclid I. 16.) is obtuse, since the angle RIS is right : the consequence finally is (by Corollary II. to Proposition III.) that the side AO is greater than the side IR.

But (OQ joined) the side AQ is greater (by Euclid I. 19.) than the side AQ, because of the obtuse angle at O, since the angle AOS was made right.

Therefore the straight AQ will be much greater than the straight IR, or (by Euclid I. 26.) than the straight MC, and so much greater than the straight AB, the part than the whole ; which is absurd.

Therefore it is not posssible to assign any one angle MNC, than which always is greater, or anyhow not less, the excess of each of the aforesaid obtuse angles above a right angle.

Wherefore those obtuse angles, more and more distant from this point A, ever more without any certain limit approach to equality with a right angle.

Quod erat postremo loco demonstrandum.

Corollary. But this standing, which in the last case was demonstrated, it manifestly follows that those straights AX, BX, produced infinitely will finally have, either in two distinct points, or in one same point X infinitely distant, a common perpendicular.

But again, that this common perpendicular cannot be had in two distinct points flows manifestly from this, because otherwise (by Corollary II. to Proposition XXIII.) those straights would thence begin mutually to separate, and so not meet each other at an infinite distance ; so that also (against the express supposition) they would not mutually approach each other without any certain limit ever more toward those parts.

So they must have the common perpendicular in one same point X infinitely distant.

[To be Continued.]